Defense Efficiency

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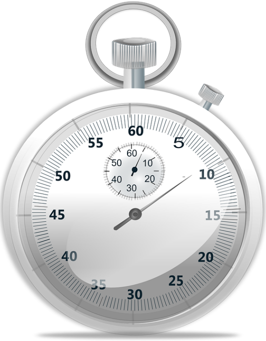
Measure of Efficiency: Why?

While defending against attacks, we need to

- evaluate the defenses.
- learn (ML)
- etc.

To this end, we need to define efficiency of defense.



Measure of Efficiency: Which?

It is important that the efficiency will

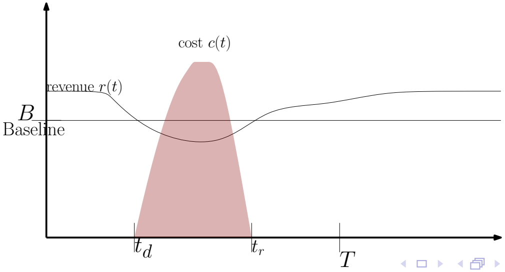
- reflect the cost-benefit tradeoff
- be higher when the system recovers

Approach

1. formally model
2. look for a useful sets of properties
3. suggest a natural satisfying solution
4. add properties to characterize the solution
5. Generalize the solution

Model

1. Revenue $r\colon \mathbb{R} \to \mathbb{R}_+$
2. Time bound T
3. Detection and recovery times t_d and t_r relatively to B
4. Impact $I \triangleq \int_{t_d}^{t_r \text{ or } T} (B - r(t))dt$
5. Cost $c\colon \mathbb{R} \to \mathbb{R}_+$
6. Total cost $Ct \triangleq \int_{t_d}^{t_r \text{ or } T} (c(t))dt$



Required Properties

- Decreasing with impact I
- Decreasing with total cost Ct
- No recovery is always smaller than recovery
- $E\colon \{\text{recovered, not recovered}\} \times \mathbb{R}_+ \times \mathbb{R}_+ \to [0,1]$

Natural Definition

Let C bound the cost, β divide recovery from no recovery and $\alpha \in [0, 1-\beta]$ define the importance of the impact

Definition

Define $E(\textit{recovered or not}, I, Ct)$ *as*

$$\begin{cases} \beta + \alpha\frac{B\cdot T - I}{B\cdot T} + (1-\beta-\alpha)\frac{C\cdot T - Ct}{C\cdot T} \\ = 1 - \frac{\alpha}{B\cdot T}I - \frac{1-\beta-\alpha}{C\cdot T}Ct & \textit{Recovered}, \\ \alpha(\frac{\beta}{1-\beta})\frac{B\cdot T - I}{B\cdot T} + (1-\beta-\alpha)(\frac{\beta}{1-\beta})\frac{C\cdot T - Ct}{C\cdot T} \\ = \beta - \alpha\frac{\beta}{(1-\beta)(B\cdot T)}I - (1-\beta-\alpha)\frac{\beta}{(1-\beta)(C\cdot T)}Ct & \textit{otherwise}. \end{cases} \quad (1)$$

This equation has the above properties.

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Axiomatic Characterization Theorem

$$\begin{cases} \beta + \alpha\frac{B\cdot T - I}{B\cdot T} + (1-\beta-\alpha)\frac{C\cdot T - Ct}{C\cdot T} \\ = 1 - \frac{\alpha}{B\cdot T}I - \frac{1-\beta-\alpha}{C\cdot T}Ct & \text{Recovered,} \\ \alpha(\frac{\beta}{1-\beta})\frac{B\cdot T - I}{B\cdot T} + (1-\beta-\alpha)(\frac{\beta}{1-\beta})\frac{C\cdot T - Ct}{C\cdot T} \\ = \beta - \alpha\frac{\beta}{(1-\beta)(B\cdot T)}I - (1-\beta-\alpha)\frac{\beta}{(1-\beta)(C\cdot T)}Ct & \text{otherwise.} \end{cases}$$

is the unique definition that satisfies the following:

1. Linearly decreasing with I
2. Linearly decreasing with Ct
3. The ratio of the linear coefficient of the impact to the linear coefficient of the total cost is independent of the recovery
4. If no recovery takes place, exactly all the values in $[0, \beta]$ are obtained; otherwise, exactly the values in $[\beta, 1]$ are obtained

Generalization to More Inputs

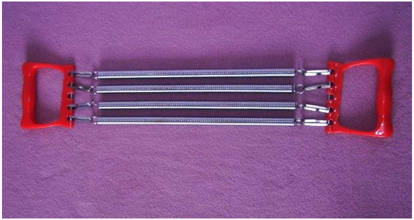
- More variables (e.g., multiple revenues and costs)
- Each variable has positive or negative influence
- Want to arrive at a measure between 0 and 1

Generalization: Expanding Equation

Required Properties

Let f be a strictly increasing function

- Increasing with each $f(y_i), i = 1, \ldots, m$
- Decreasing with each $f(x_j), j = m+1, \ldots, m+l$
- No recovery is always smaller than recovery
- $E: \{\text{recovered, not recovered}\} \times \mathbb{R}_+^{m+l} \to [0, 1]$



Expanding Equation: Natural Definition

Let nonnegative $\alpha_1, \alpha_2, \ldots, \alpha_{m+l-1}$ s.t. $\sum_{i=1}^{m+i-1} \alpha_i \leq 1 - \beta$ define the importance of each term. Let each y_i be bounded by Y_i and let each x_j be bounded by X_j.

Definition

Define $E(\textit{recovered or not}, y_1, \ldots, y_m, x_{m+1}, \ldots, x_{m+l})$ *as*

$$\begin{cases} \beta + \sum_{i=1}^{m} \alpha_i \frac{f(y_i)}{f(Y_i)} + \sum_{j=m+1}^{m+l-1} \alpha_j \frac{f(X_j)-f(x_j)}{f(X_j)} \\ +(1 - \beta - \sum_{k=1}^{m+l-1} \alpha_k) \frac{f(X_{m+l})-f(x_{m+l})}{f(X_{m+l})} & \textit{Recovered}, \\ \sum_{i=1}^{m} \alpha_i (\frac{\beta}{1-\beta}) \frac{f(y_i)}{f(Y_i)} + \sum_{j=m+1}^{m+l-1} \alpha_j (\frac{\beta}{1-\beta}) \frac{f(X_j)-f(x_j)}{f(X_j)} \\ +(1 - \beta - \sum_{k=1}^{m+l-1} \alpha_k)(\frac{\beta}{1-\beta}) \frac{f(X_{m+l})-f(x_{m+l})}{f(X_{m+l})} & \textit{otherwise}. \end{cases} \tag{2}$$

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Definition

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Expanding Equation: Axiomatic Characterization Theorem

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4. If no recovery takes place, exactly the values in $[0, \beta]$ can be obtained; otherwise, all the values in $[\beta, 1]$ and only they can be obtained

Generalization: Combining Efficiency Values

Definition

Given E_i, the ith component of the efficiency, and its weight γ_i, define

$$E \triangleq \sum_{i=1}^{n} \gamma_i E_i, \tag{3}$$

where $\gamma_i \geq 0$, $\sum_{i=1}^{n} \gamma_i = 1$.

Each $E_i \in [0, 1] \Rightarrow E \in [0, 1]$.

Generalization: Expanding vs Combining

The two generalizations are not equivalent.
However,

Theorem

If the system recovers, they are equivalent!

Conclusions

1. Defined efficiency that is monotone and respects recovery
2. Axiomatic characterization

Generalizations:

- Expanding equation and characterizing
- Combining efficiency values as black boxes

Comparing these generalizations

Thank You!